ET 438a Automatic Control Systems Technology

LESSON 21: METHODS OF SYSTEM ANALYSIS

LEARNING OBJECTIVES

After this presentation you will be able to:

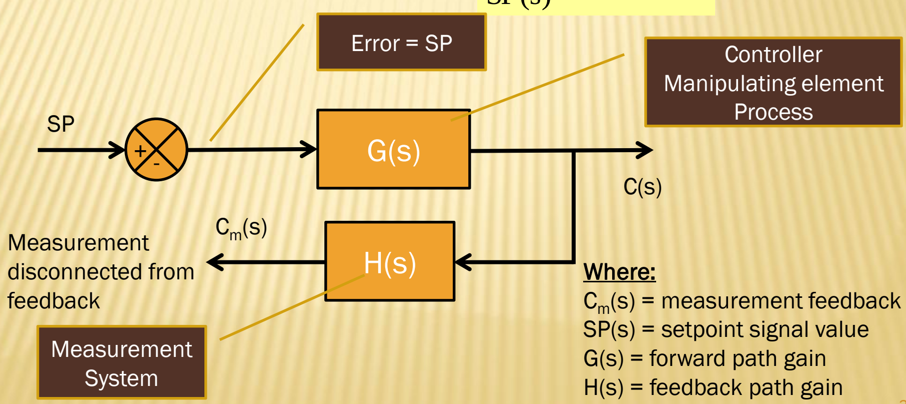
- Compute the value of transfer function for given frequencies.
- Compute the open loop response of a control system.
- Compute and interpret the closed loop response of a control system.
- Compute and interpret the error ratio of a control system.

FREQUENCY RESPONSE OF CONTROL SYSTEMS

Control limits determined by comparing the open loop response of system to closed loop response.

Open loop response of control system:

$$\frac{C_m(s)}{SP(s)} = G(s) \cdot H(s)$$

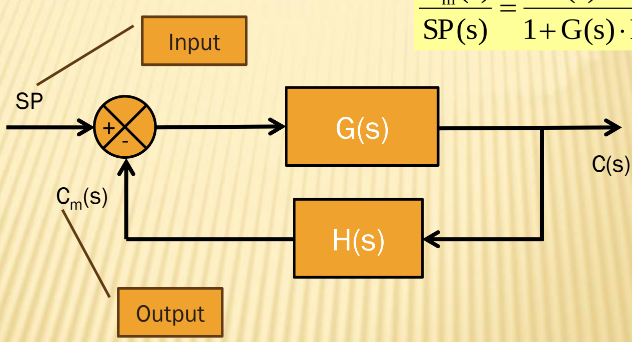


Where:
$C_m(s)$ = measurement feedback
SP(s) = setpoint signal value
G(s) = forward path gain
H(s) = feedback path gain

FREQUENCY RESPONSE OF CONTROL SYSTEMS

Closed loop response of control system:

$$\frac{C_m(s)}{SP(s)} = \frac{G(s) \cdot H(s)}{1 + G(s) \cdot H(s)}$$



Note: this is not the I/O relationship that was used earlier. (C(s)/SP(s))

FREQUENCY RESPONSE OF CONTROL SYSTEMS

Frequency response of system divided into three ranges:

Zone 1- controller decreases error
Zone 2 - controller increases error
Zone 3 - controller has no effect on error

SP change frequency determines what zone is activated. Overall system frequency determines values of zone transition frequencies

Error Ratio (ER) plot determines where zones occur

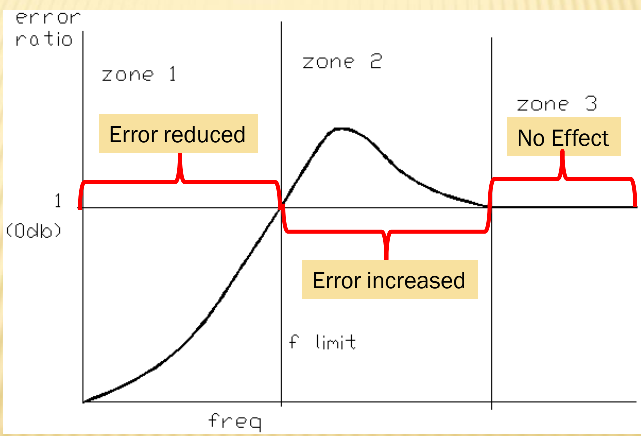
$$ER = \frac{|\text{Closed}-\text{loop Error Magnitude}|}{|\text{Open-loop Error Magnitude}|}$$

$$ER = \left|\frac{1}{[1+G(s)\cdot H(s)][1-G(s)\cdot H(s)]}\right|$$

Replace s with $j\omega$ and compute magnitude of complex number

FREQUENCY RESPONSE OF CONTROL SYSTEMS

Typical Error Ratio plot showing operating zones and controller action



COMPUTING TRANSFER FUNCTION VALUES

Example 21-1: Given the forward gain, G(s), and the feedback system gain, H(s) shown below, find 1) open loop transfer function, 2) closed loop transfer function, 3) error ratio.

$$G(s) = \frac{21.8}{1 + 0.379 \cdot s + 0.0063 \cdot s^2}$$

$$H(s) = \frac{0.356}{1 + 0.478 \cdot s}$$

4) compute the values of the open/closed loop transfer functions when ω=0.1, 1, 10 and 100 rad/sec. 5) compute the value of the error ratio when ω=0.1, 1, 10 and 100 rad/sec. 6) Use MatLAB to plot the open and closed loop transfer function responses on the same axis.

EXAMPLE 21-1 SOLUTION (1)

1) Open Loop Transfer Function

$$G(s)H(s) = \left[\frac{21.8}{1+0.379s+0.0063s^2}\right]\left[\frac{0.356}{1+0.478s}\right]$$

Expand the denominator

$$\left(1+0.379s+0.0063s^2\right)\left(1+0.478s\right)$$

$$1+0.379s+0.0063s^2+0.478s+0.1812s^2+3.01\times10^{-3}s^3$$

$$1+0.857s+0.18746s^2+0.00301s^3$$

$$G(s)H(s) = \boxed{\frac{7.761}{1+0.857s+0.18746s^2+0.00301s^3}}$$

Ans

EXAMPLE 21-1 SOLUTION (2)

2) Find closed loop transfer function

$$\frac{G(s)H(s)}{1+G(s)H(s)} = \frac{\dfrac{7.761}{1+0.857s+0.18746s^2+0.00301s^3}}{1+\left[\dfrac{7.761}{1+0.857s+0.18746s^2+0.00301s^3}\right]}$$

Multiply numerator and denominator by $1+0.857s+0.18746s^2+0.00301s^3$ and simplify

$$\frac{G(s)H(s)}{1+G(s)H(s)} = \frac{7.761}{1+0.857s+0.18746s^2+0.00301s^3+7.761}$$

$$\frac{G(s)H(s)}{1+G(s)H(s)} = \boxed{\frac{7.761}{8.761+0.857s+0.18746s^2+0.00301s^3}}$$

Ans

EXAMPLE 21-1 SOLUTION (3)

3) Find the error ratio

$$ER(s) = \left|\left(\frac{1}{1+G(s)H(s)}\right)\left(\frac{1}{1-G(s)H(s)}\right)\right|$$

Magnitude only
Expand denominator and simplify

$$[1+G(s)H(s)][1-G(s)H(s)] = 1-\cancel{G(s)H(s)}+\cancel{G(s)H(s)}-[G(s)H(s)]^2$$

$$[1+G(s)H(s)][1-G(s)H(s)] = 1-[G(s)H(s)]^2$$

Substitute in G(s)H(s) from part 1 into above

$$ER(s) = \frac{1}{1-\left[\dfrac{7.761}{1+0.867s+0.18746s^2+0.00301s^3}\right]^2}$$

EXAMPLE 21-1 SOLUTION (4)

Error ratio calculations

$$ER(s)=\frac{1}{1-\dfrac{7.761^2}{(1+0.867s+0.18746s^2+0.003015s^3)^2}}$$

$$ER(s)=\frac{(1+0.867s+0.18746s^2+0.003015s^3)^2}{(1+0.867s+0.18746s^2+0.003015s^3)^2+7.761^2}$$

$$ER(s)=\boxed{\frac{(1+0.867s+0.18746s^2+0.003015s^3)^2}{(1+0.867s+0.18746s^2+0.003015s^3)^2-60.23}}$$

Ans

EXAMPLE 21-1 SOLUTION (5)

4) Compute the values of the open and closed loop transfer functions for ω=0.1 1 10 100 rad/s Substitute jω for s

Open loop

$$G(j\omega)=\frac{21.8}{1+0.379j\omega+0.0063(j\omega)^2}$$

NOTE: $j^2=-1$
$j^3=-j$

$$H(j\omega)=\frac{0.356}{1+0.478j\omega}$$

$$GH(j\omega)=G(j\omega)H(j\omega) \quad \omega=0.1$$

$$G(j0.1)=\frac{21.8}{1+0.379(j0.1)+0.0063(j0.1)^2}=\frac{21.8}{1+0.0379j-0.000063}=\frac{21.8}{0.999+0.0379j}$$

EXAMPLE 21-1 SOLUTION (6)

$$H(j0.1) = \frac{0.356}{1+0.478j0.1} = \frac{0.356}{1+0.0478j} = 0.355\angle{-2.74^\circ}$$

$$G(j0.1) = 21.79 - j0.827 = 21.81\angle{-2.17^\circ}$$

$$G(j0.1)H(j0.1) = (21.81\angle{-2.17^\circ})(0.355\angle{-2.74^\circ})$$

$$G(j0.1)H(j0.1) = \boxed{7.74\angle{-4.91^\circ}}$$

Now for ω=1 rad/sec

$$G(j1) = \frac{21.8}{1+0.379j1+0.0063(j1)^2} = \frac{21.8}{1+0.379j-0.0063} = \frac{21.8}{0.994+0.379j}$$

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EXAMPLE 21-1 SOLUTION (7)

$$G(j1) = 19.15 - 7.301j = 20.49\angle{-20.87^\circ}$$

$$H(j1) = \frac{0.356}{1+0.478j1} = \frac{0.356}{1+0.478j} = 0.322\angle{-25.61^\circ}$$

$$G(j1)H(j1) = (20.49\angle{-20.87^\circ})(0.322\angle{-25.61^\circ}) = \boxed{6.598\angle{-46.48^\circ}}$$

Now for ω=10 rad/sec

$$G(j10) = \frac{21.8}{1+0.379j10+0.0063(j10)^2} = \frac{21.8}{1+3.79j-0.63} = \frac{21.8}{0.37+3.79j}$$

EXAMPLE 21-1 SOLUTION (8)

$$G(j10) = 0.556 - 5.698j = 5.725\angle{-84.43^\circ}$$

$$H(j10) = \frac{0.356}{1+0.478j10} = \frac{0.356}{1+4.78j} = 0.073\angle{-78.1^\circ}$$

$$G(j100) = \frac{21.8}{1+0.379j100+0.0063(j100)^2} = \frac{21.8}{1+37.9j-63} = \frac{21.8}{-62+37.9j}$$

For ω=100 rad/sec

$$G(j100) = \frac{21.8}{1+0.379j100+0.0063(j100)^2} = \frac{21.8}{1+37.9j-63} = \frac{21.8}{-62+37.9j}$$

EXAMPLE 21-1 SOLUTION (9)

$$G(j100) = -0.256 - 0.156j = 0.3\angle{-148.64^\circ}$$

$$H(j100) = \frac{0.356}{1+0.478j100} = \frac{0.356}{1+47.8j} = 1.56\times10^{-4} - 7.44\times10^{-3}j$$

$$H(j100) = 7.446\times10^{-3}\angle{-88.8^\circ}$$

$$G(j100)H(j100) = (0.3\angle{-148.64^\circ})(7.446\times10^{-3}\angle{-88.8^\circ}) = \boxed{0.00223\angle{122.6^\circ}}$$

EXAMPLE 21-1 SOLUTION (10)

Convert all gain values into dB

$\omega=0.1$	$20\,Log(7.746) = 17.78\,dB$	-4.91° Phase
$\omega=1$	$20\,Log(6.598) = 16.37\,dB$	-46.4° Phase
$\omega=10$	$20\,Log(0.918) = -7.59\,dB$	-162.5° Phase
$\omega=100$	$20\,Log(0.00223) = -53.0\,dB$	127.6° Phase

4) Compute the close loop response using the previously calculated values of G(s)H(s)

Define $GH_c(s) = \dfrac{G(s)H(s)}{1+G(s)H(s)}$ and $GH_c(j\omega) = \dfrac{G(j\omega)H(j\omega)}{1+G(j\omega)H(j\omega)}$

$j\omega = 0.1j \qquad G(j0.1)H(j0.1) = 7.74\angle -4.91^\circ$

EXAMPLE 21-1 SOLUTION (11)

$$GH_c(j0.1) = \frac{7.74\angle -4.91^\circ}{1+7.74\angle -4.91^\circ}$$

$$GH_c(j0.1) = \boxed{0.886\angle -0.56^\circ}$$

$j\omega = 1j \qquad G(j1)H(j1) = 6.598\angle -46.48^\circ$

$$GH_c(j1) = \frac{6.598\angle -46.48^\circ}{1+6.598\angle -46.48^\circ}$$

$$GH_c(j1) = \boxed{0.901\angle -5.68^\circ}$$

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EXAMPLE 21-1 SOLUTION (12)

$$j\omega = 10j \qquad G(j10)H(j10) = 0.418\angle{-162.53^\circ}$$

$$GH_c(j10) = \frac{0.418\angle{-162.53^\circ}}{1 + 0.418\angle{-162.53^\circ}}$$

$$GH_c(j10) = \boxed{0.681\angle{-150.74^\circ}}$$

$$j\omega = 100j \qquad G(j100)H(j100) = 0.00223\angle{127.6^\circ}$$

$$GH_c(j100) = \frac{0.00223\angle{127.5^\circ}}{1 + 0.00223\angle{127.5^\circ}}$$

$$GH_c(j100) = \boxed{0.00223\angle{127.5^\circ}}$$

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EXAMPLE 21-1 SOLUTION (13)

Convert all gain values into dB

$$20\,\mathrm{Log}(GH_c(j0.1)) = 20\,\mathrm{Log}(0.886) = -1.051\text{ dB} \quad -0.56^\circ\text{ Phase}$$

$$20\,\mathrm{Log}(GH_c(j1)) = 20\,\mathrm{Log}(0.901) = -0.911\text{ dB} \quad -5.68^\circ\text{ Phase}$$

$$20\,\mathrm{Log}(GH_c(j10)) = 20\,\mathrm{Log}(0.681) = -3.36\text{ dB} \quad -150.7^\circ\text{ Phase}$$

$$20\,\mathrm{Log}(GH_c(j100)) = 20\,\mathrm{Log}(0.00223) = -53\text{ dB} \quad +127.5^\circ\text{ Phase}$$

5) Compute the values of the error ratio

$$ER = \left| \frac{1}{[1+G(s)H(s)][1-G(s)H(s)]} \right|$$

Use open loop values to compute values of ER at given frequencies

EXAMPLE 21-1 SOLUTION (14)

At $j\omega = j0.1$ $\quad G(j\omega)H(j\omega) = 7.746\angle{-49.1^\circ}$

$$ER = \left|\frac{1}{(1+7.746\angle{-49.1^\circ})(1-7.746\angle{-49.1^\circ})}\right|$$

$$ER = \left|\frac{1}{59.015\angle{170^\circ}}\right| = \boxed{0.017}$$

Now for ω=1 rad/sec

$$G(j\omega)H(j\omega) = 6.598\angle{-46.48^\circ}$$

EXAMPLE 21-1 SOLUTION (15)

$$ER = \left|\frac{1}{(1+6.598\angle{-46.48^\circ})(1-6.598\angle{-46.48^\circ})}\right|$$

$$ER = \left|\frac{1}{43.41\angle{85.9}}\right| = \boxed{0.023}$$

For ω=10 rad/sec $\quad G(j\omega)H(j\omega) = 0.00223\angle{127.5^\circ}$

$$ER = \left|\frac{1}{(1+0.00223\angle{127.5^\circ})(1-0.00223\angle{127.5^\circ})}\right|$$

$$ER = \left|\frac{1}{1\angle{2.62^\circ}}\right| = \boxed{1}$$

EXAMPLE 21-1 SOLUTION (16)

Convert all gain values into dB

$j\omega = j0.1 \quad 20\,Log(0.017) = -35.4\,dB$

$j\omega = j1 \quad 20\,Log(0.023) = -32.8\,dB$

$j\omega = j10 \quad 20\,Log(1.159) = 1.28\,dB$

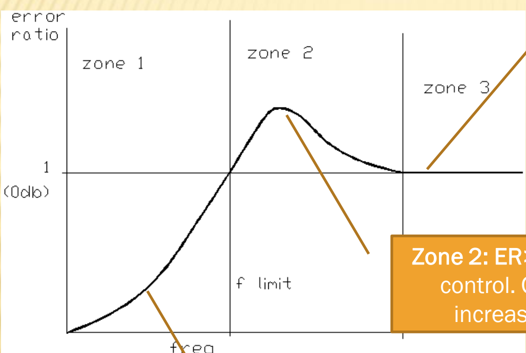
$j\omega = j100 \quad 20\,Log(1) = 0\,dB$

System becomes uncontrollable between these two frequencies

Error ratio magnitude increases as frequency increases. It peak and becomes a constant value of 1 (0 dB)

INTERPRETING ERROR RATIO PLOTS

Define control zones



Zone 3: ER-=0 dB no control. Controller does not change error

Zone 2: ER> 0 dB poor control. Controller increases error

Zone 1: ER< 0 dB good control. Controller decreases error

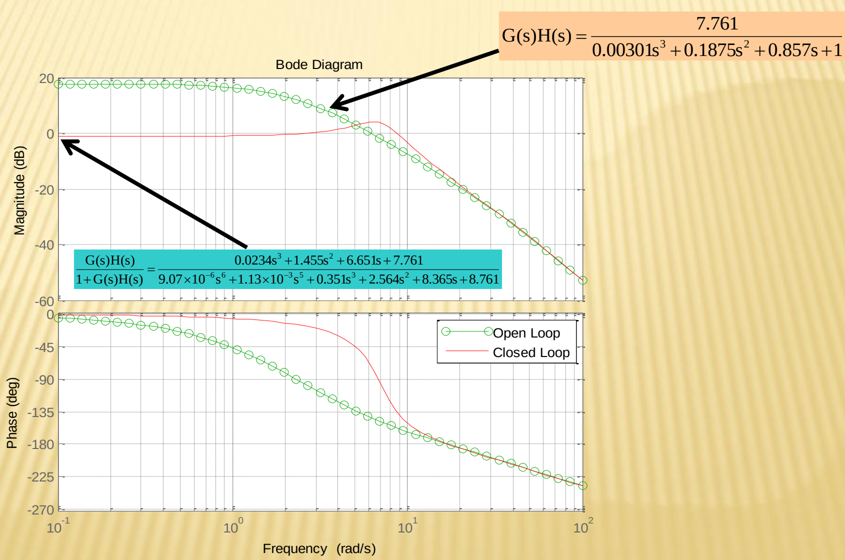
GENERATING PLOTS USING MATLAB

Use MatLAB script to create open and closed loop Bode plots of example system

```
% Example bode calculations
clear all;
close all;
% define the forward gain numerator and denominator coefficients
numg=[21.8];
demg=[0.0063 0.379 1];
% define the feedback path gain numerator and denominators
numh=[0.356];
demh=[0.478 1];
% construct the transfer functions
G=tf(numg,demg);
H=tf(numh,demh);
% find GH(s)
GH=G*H
% find the closed loop transfer function
GHc=GH/(1+GH)

% The value in curly brackets are freq. limits
bode(GH,'go-',GHc,'r-‘,{0.1,100})
```

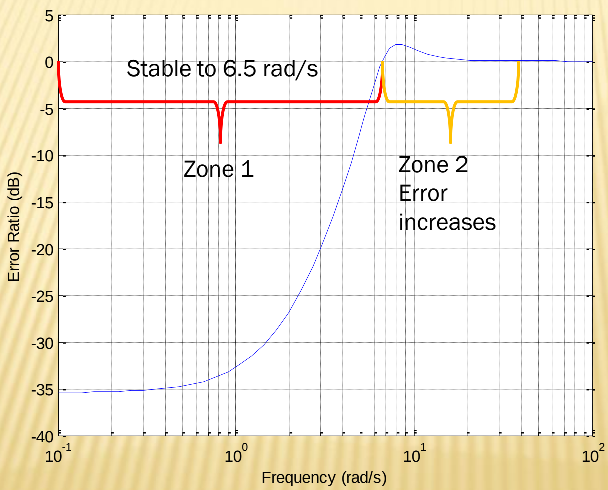

BODE PLOTS OF EXAMPLE 21-1



MATLAB CODE FOR ERROR PLOT EXAMPLE 21-1

```
% Example Error Ratio calculations
clear all;
close all;
% define the forward gain numerator and denominator coefficients
numg=[21.8];
demg=[0.0063 0.379 1];
% define the feedback path gain numerator and denominators
numh=[0.356];
demh=[0.478 1];
% construct the transfer functions
G=tf(numg,demg);
H=tf(numh,demh);
% find GH(s)
GH=G*H;
% find the error ratio
 ER=1/((1+GH)*(1-GH));
 [mag,phase,W]=bode(ER,{0.1,100});  %Use bode plot with output sent to arrays
N=length(mag);  %Find the length of the array
gain=mag(1,1:N);  %Extract the magnitude from the mag array
db=20.*log10(gain);  % compute the gain in dB and plot on a semilog plot
semilogx(W,db);
grid on;    %Turn on the plot grid and label the axis
xlabel('Frequency (rad/s)');
ylabel('Error Ratio (dB)');
```


ERROR RATIO PLOT



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END LESSON 21: METHODS OF SYSTEM ANALYSIS